Math 121 — Test #3 Answers — Fall 2011

1. (12 pts) This is a one-sample test of a proportion. Here are the seven steps.

   (1) Let p represent the proportion of Virginians who believe that their pets will join them in heaven.
   $H_0: \quad p = 0.50$
   $H_1: \quad p < 0.50$

   (2) $\alpha = 0.10$.

   (3) Let $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

   (4) In the sample, we see that $\hat{p} = 0.48$. Compute z.

   $$\begin{aligned} z &= \frac{0.48 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{600}}} \\ &= -\frac{0.02}{0.0204} \\ &= -0.9798. \end{aligned}$$

   (5) The direction of extreme is to the left.

   $$\begin{aligned} p\text{-value} &= \texttt{normalcdf(-E99,-.9798)} \\ &= 0.1636. \end{aligned}$$

   (6) The p-value is greater than α, so we should accept H_0.

   (7) The proportion of Virginians who believe that their pets will join them in heaven is not less than 50%.

   You can work steps (4) and (5) on the TI-83 using the `1-PropZTest` function. Enter 0.50 for `p`$_0$, 48% of 600 (i.e., 288) for `x`, 600 for `n`, and choose `<p`$_0$ for the test. The calculator reports that $z = -0.9798$ and that the p-value is 0.1636.

2. (10 pts)

   (a) (8 pts) The confidence interval is

   $$\begin{aligned} \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.48 \pm 1.96\sqrt{\frac{(0.48)(0.52)}{600}} \\ &= 0.48 \pm 0.0400. \end{aligned}$$

   You can use the `1-PropZInt` function on the TI-83. Enter 288 for `x`, 600 for `n`, and 0.95 for `C-Level`. We get the interval $(0.44002, 0.51998)$.

   (b) (2 pts) The margin of error is 0.0400. If you used the TI-83 in part (a), then you need to take half the difference between the endpoints or take the difference between 0.51998 and $\hat{p}$. Either way, you get 0.0400.

3. (14 pts)

   (a) (2 pts) We should use a t test. The sample size is small (< 30) and we are using s instead of σ. It is also relevant whether the underlying population is normal, but that was not mentioned in the problem. (Ooops!)

   (b) (12 pts) The seven steps:

      (1) Let μ represent average net worth of young people.
      $H_0: \quad \mu = 0$
      $H_1: \quad \mu > 0$

      (2) $\alpha = 0.05$.

      (3) In part (a) we decided to use the t statistic. Let $t = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$.

      (4) We have $\overline{x} = 3600$ and $s = 9500$. We compute

$$\begin{aligned} t &= \frac{3600 - 0}{9500/\sqrt{25}} \\ &= \frac{3600}{1900} \\ &= 1.8947. \end{aligned}$$

      (5) There are 24 degrees of freedom (1 less than n). The p-value is

$$\begin{aligned} p\text{-value} &= \texttt{tcdf(1.8947,E99,24)} \\ &= 0.03512. \end{aligned}$$

      (6) The p-value is less than α, so we should reject H_0.

      (7) We conclude that the average net worth of young people is greater than 0.

   You can work steps (4) and (5) on the TI-83 using the `T-Test` function with the `Stats` option. Enter 0 for μ_0, 3600 for $\overline{\texttt{x}}$, 9500 for `Sx`, 25 for `n`, and choose $> \mu_0$ for the test. The calculator reports that $t = 1.8947$ and that the p-value is 0.03512.

4. (14 pts) This is a two-sample test of proportions. The seven steps are:

   (1) Let p_1 represent the breast cancer rate among women who do not consume alcohol and let p_2 be the breast cancer rate among women who consume three to six drinks a week.
   $H_0: \quad p_1 = p_2$
   $H_1: \quad p_1 < p_2$

   (2) $\alpha = 0.05$.

   (3) The test statistic is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

   where $\hat{p}$ is the pooled estimate for p.

(4) We have $\hat{p}_1 = \frac{24}{1000} = 0.024$, $\hat{p}_2 = \frac{40}{1000} = 0.040$, and $n_1 = n_2 = 1000$. The pooled estimate for p is $\hat{p} = \frac{24+40}{1000+1000} = 0.032$. We calculate

$$\begin{aligned} z &= \frac{(0.024 - 0.040) - 0}{\sqrt{(0.032)(0.968)\left(\frac{1}{1000} + \frac{1}{1000}\right)}} \\ &= -\frac{0.016}{0.007871} \\ &= -2.0328. \end{aligned}$$

(5) The p-value is

$$\begin{aligned} p\text{-value} &= \texttt{normalcdf(-E99,-2.0328)} \\ &= 0.0210. \end{aligned}$$

(6) The p-value is less than α, so reject H_0.

(7) We conclude that the breast cancer-rate among women who do not consume alcohol is less than the breast-cancer rate among women who consume three to six drinks a week.

You can work steps (4) and (5) on the TI-83 using the `2-PropZTest` function. Enter 24 for $\mathtt{x}_1$, 1000 for $\mathtt{n}_1$, 40 for $\mathtt{x}_2$, 1000 for $\mathtt{n}_2$, and choose $\mathtt{<p}_2$ for the test. The calculator reports that $z = -2.0328$ and that the p-value is 0.0210.

5. (9 pts)

   (a) (3 pts) `tcdf(-E99,1.4,2)` = 0.8518.

   (b) (3 pts) `tcdf(-E99,1.4,10)` = 0.9041.

   (c) (3 pts) `tcdf(-1,1,30)` = 0.6747.

6. (12 pts)

   (a) (3 pts) Two dependent (paired) samples. The values for the husbands and the wives are paired, but there is a sample of husbands and a dependent sample of wives.

   (b) (3 pts) Two independent samples. There is no logical or meaningful way to pair the shots taken by Jim with the shots taken by Joe.

   (c) (3 pts) Two dependent samples. The values from the two thermometers are paired when they are taken at the same time.

   (d) (3 pts) There is only one sample of coin tosses.

7. (5 pts) The pooled estimate is $\hat{p} = \frac{12+26}{25+40} = \frac{38}{65} = 0.5846$.

8. (10 pts)

   (a) (5 pts) The margin of error is half the width of the interval, or 0.10.

(b) (5 pts) The margin of error will increase. In fact, the larger margin of error is the reason for the higher level of confidence.

9. (14 pts) This is a two-sample test of means. The seven steps are:

(1) Let μ_1 represent the average number of visits to the doctor for people living near the industrial park and let μ_2 represent the average number of visits to the doctor for people living far from the industrial park.
$H_0: \quad \mu_1 = \mu_2$
$H_1: \quad \mu_1 > \mu_2$

(2) $\alpha = 0.01$.

(3) Both sample sizes are large, so may use z, but I'll use t. The test statistic is

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - 0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$$

where s_p is the pooled estimate for σ.

(4) We have $\overline{x}_1 = 6.4$, $s_1 = 1.6$, $\overline{x}_2 = 5.2$, $s_2 = 1.2$, $n_1 = 60$, and $n_2 = 40$. The pooled estimate for σ is

$$\begin{aligned} s_p &= \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \\ &= \sqrt{\frac{59 \cdot (1.6)^2 + 39 \cdot (1.2)^2}{98}} \\ &= 1.4540. \end{aligned}$$

We calculate

$$\begin{aligned} t &= \frac{(6.4 - 5.2) - 0}{1.4540\sqrt{\frac{1}{60} + \frac{1}{40}}} \\ &= \frac{1.2}{0.2968} \\ &= 4.0430. \end{aligned}$$

(5) The p-value is

$$\begin{aligned} p\text{-value} &= \texttt{tcdf(4.0430,E99,98)} \\ &= 5.2583 \times 10^{-5}. \end{aligned}$$

(6) The p-value is less than α, so reject H_0.

(7) We conclude that the average number of visits to the doctor annually is greater among those who live near the industrial park than it is among those who live far from the park.

You can work steps (4) and (5) on the TI-83 using the `2-SampTTest` function using the `Stats` option. Enter 6.4 for $\overline{\mathtt{x}}_1$, 1.6 for $\mathtt{Sx}_1$, 60 for n_1, 5.2 for $\overline{\mathtt{x}}_2$, 1.2 for $\mathtt{Sx}_2$, 40 for n_2, and choose $\mathtt{>}\mu_2$ for the test. The calculator reports that $t = 4.0430$ and that the p-value is 5.2583×10^{-5}.